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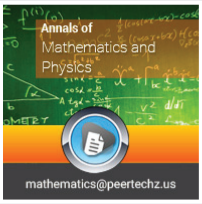

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Short Communication

Fractal space-time and variations of the hubble constant

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Abstract

Spatial variations of the Hubble constant are considered according to Riess, et al. (2018). It is noted that the values of the Hubble constant form an almost fractal manifold. This fact suggests that the variations may be associated with local gravitational perturbations in the neighborhoods of galaxies, in which there are Cepheids and supernovae selected for measurement. The aim and purpose of the study is to show that the spatial variations of the Hubble constant may be due to the fact that the galaxies belong to outskirts of the Local Supercluster.

Introduction

Edwin Powell Hubble's article on "a relation between distance and radial velocity among extra-galactic nebular" was published 94 years ago [1]. It is believed that Hubble's discovery was the first law of the observational cosmology. Now Hubble's law is considered the first observational basis for the cosmic space expansion.

The notion of the cosmic space expanding was first derived from the General Relativity (GR) equations in 1922 by Alexander Friedmann in his article "On the Possibility of a World with Constant Negative Curvature of Space" [2]. In the present-day cosmology the Friedmann-Lemaitre-Robertson-Walker (FLRW) model or a homogeneous and isotropic cosmological model is the basis of the standard cosmology LCDM paradigm. The Hubble constant H_o is the most important cosmological parameter. It is used to estimate cosmological distances to galaxies, their clusters. It is used in the theoretical cosmology through the Hubble parameter $H=\frac{\dot{a}}{a}$, where a is the scale factor of the cosmological model and a dot above the letter means differentiation according to the cosmological time t of the model. In the FLRW model the Hubble parameter depends only on cosmological time, and in the modern era t_o, the Hubble constant $H(t_0)=H_0$ has to be identical in all directions.

The results of the latest local measurements H_o were very carefully performed in [3,4] using data on Cepheids in 23 galaxies, supernovae SN Ia were observed in 19 galaxies (the redshifts $0{,}01<z<0{,}15$ observations by the Hubble Space Telescope HST). The average value is equal to $H_0^{HST}=(73{,}48\pm1{,}66)\,km/s/Mpc$ for the standard cosmological model.

Global estimates of the Hubble constant are obtained by fitting cosmological parameters of ΛCDM model to observations of CMB anisotropy and galaxy clustering. According to the latest data from the Planck Collaboration [5] for a spatially flat model we have $H_0^{Pl}=(67{,}4\pm0{,}5)\,km/s/Mpc$.

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In recent publications, an unexpectedly large difference between H_0^{HST} and H_0^{Pl} is explained by the possibility of the presence of still unclear errors of measurements or hypotheses about the dark energy model (for example, [6]) or about the inhomogeneity of dark matter distribution (for example, [7,8]).

Here I propose a simpler hypothesis to explain inequality $H_0^{HST} \neq H_0^{Pl}$ remaining within the GR framework and without the use of exotic forms of matter. The hypothesis itself appeared due to a careful look at the measurement data H_0^{HST} in various galaxies [3]. A sample of these data is shown in Table 1. It appears, the measured values H_0^{HST} form a manifold that is approximately described by the power low characterizing fractals. It has the following form $H_0^{HST} \approx H^*(1,023)^n$. Here $H^* = 65 km/s/Mpc$, and the values of the exponent n are shown in Table 1.

Let's suppose that this form reflects the real local properties of the cosmic space in the vicinity of supernovae (or galaxies) that were used for the measurement. Variations of the value H_0^{HST} relative to the value H_0^{Pl} indicate the presence of local gravitational perturbations in the vicinity of each source: local changes of the space-time metric relative to the metric of background reference system.

Table 1: Hubble constant (data [3]).

Galaxy	H_0^{HST} (km/s/Mpc)	*n*
M101	68.39	2
N1015	80.09	9
N1309	70.24	3
N1365	68.39	2
N1448	77.77	8
N2442	73.42	5
N3021	63.94	-1
N3370	76.00	7
N3447	74.37	6
N3972	77.98	8
N3982	64.80	-1
N4038	79.69	9
N4258	72.25	5
N4424	63.97	-1
N4536	71.48	4
N4639	77.98	8
N5584	78.67	9
N5917	72.75	5
N7250	74.75	6
U9391	66.53	1

The fractality indicates that sources are in regions of space-time that have self-similar geometric properties. For example the regions may be are the filaments of fractal cosmic web or self-similar regions with equals intervals $ds^2 = g_{ik}dx^i dx^k$. The metric tensors of any two regions and their coordinates differ only by a constant coefficient: $(g_{ik})_m = q_{mn}\cdot(g_{ik})_n$ $(x^i)_m = \frac{1}{q_{mn}}(x^i)_n$ $q_{mn} = const$. This fractal space-time composed of such self-similar regions is statistic homogeneity. In article [9] the algorithm of a construction of the GR solutions for the fractal space-time is described.

Examples of cosmic fractals and short review of the past and recent related research work are given in the article [10].

Gravitational perturbations and hubble constant variations

Let's here consider local gravitational perturbations $h_{\alpha\beta}$ in the vicinity of the galaxy which host Cepheids and supernovae, and use the classical synchronous space-time metric:

$$ds^2 = g_{ik}dx^i dx^k = a^2\cdot d\eta^2 - \left(a^2\delta_{\alpha\beta} + h_{\alpha\beta}\right)dx^\alpha dx^\beta \quad (1)$$

Where Latin indices run through values 0, 1, 2, 3, Greek indices 1, 2, 3. The coordinates of the background reference system are a conformal time η and space coordinates x^α: $a\cdot d\eta = c\cdot dt$, c is the light speed.

The commoving distance of galaxy r_g is measured along the isotropic geodesic of space-time with metric (1). For the isotropic geodesic $ds = 0$, and $c^2\cdot dt^2 = -\left(g_{\alpha\beta} + h_{\alpha\beta}\right)u^\alpha u^\beta dr_g^{\,2}$, u^i is the isotropic wave vector. For the background reference system $u^i = \frac{k^i}{a}$, and k^i does not depend on η.

The commoving distance of galaxy r_g is equal to

$$r_g = \int_0^{z_g} u_\alpha dx^\alpha = c\int_0^{z_g} \frac{dt}{dz}\frac{u_\alpha u^\alpha}{\sqrt{-\left(g_{\alpha\beta} + h_{\alpha\beta}\right)u^\alpha u^\beta}}dz$$

$$= ck^2\int_0^{z_g} \frac{dt}{dz}\frac{dz}{a\sqrt{\left(\delta_{\alpha\beta} + \frac{h_{\alpha\beta}}{a^2}\right)k^\alpha k^\beta}} \quad (2)$$

Where $k^2 = k_\alpha k^\alpha$, $\frac{dt}{dz} = \frac{1}{H_0}\frac{1}{\sqrt{1+\Omega_m(1+z)^3 + \Omega_\Lambda}}$, Ω_m and Ω_Λ are the density parameters of the standard cosmology ΛCDM model.

Let's consider a luminosity distance of galaxy $d_L = \left(1+z_g\right)r_g$. It is used in the modulus distance equation of source:

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$$(m-M)_{PL} = 5\log\frac{d_L}{Mps}+25 \tag{3}$$

Where $(m-M)_{PL}$ is a distance module for Cepheids. The authors of [3] found values H_0^{HST} for dozens of Cepheids in each galaxy. The most probable value H_0^{HST} (included in Table 1) was determined from the condition of equality of distance modules for Cepheids and a supernova in the same galaxy.

Using equations (2) - (3), we find the expression for the Hubble constant taking into account the gravitational perturbation:

$$H_0 = ck\left(1+z_g\right)\cdot 10^{5-\frac{(m-M)_{PL}}{5}} \cdot\int_0^{z_g}\frac{1+z}{\sqrt{1+\Omega_m\left(1+z\right)^3+\Omega_\Lambda}}\frac{dz}{\sqrt{1+\frac{h_{\alpha\beta}k^\alpha k^\beta}{a^2k^2}}} \tag{4}$$

The Hubble constant H_0^{Pl} for the background frame is obtained from equation (4) with $\frac{h_{\alpha\beta}k^\alpha k^\beta}{a^2k^2}=0$. Then the relative variation of the Hubble constant is equal to

$$\frac{H_0-H_0^{Pl}}{H_0^{Pl}} \approx \frac{\frac{1}{2}\int_0^{z_g}\frac{1+z}{\sqrt{1+\Omega_m\left(1+z\right)^3+\Omega_\Lambda}}\frac{h_{\alpha\beta}k^\alpha k^\beta}{a^2k^2}dz}{\int_0^{z_g}\frac{1+z}{\sqrt{1+\Omega_m\left(1+z\right)^3+\Omega_\Lambda}}dz} \tag{5}$$

where $\frac{h_{\alpha\beta}k^\alpha k^\beta}{a^2k^2} \ll 1$

One can assume the main contribution to the integral in the numerator of the fraction (5) is given by neighborhood of the galaxy at an epoch $z=z_g$. Then, to estimate the variation of the value of the Hubble constant, two conditions are used: $\frac{h_{\alpha\beta}k^\alpha k^\beta}{a^2k^2}\approx\frac{k^\alpha k^\beta}{k^2}\left(\frac{h_{\alpha\beta}}{a^2}\right)_{z_g}$ and $h_{\alpha\beta}\left(z<z_g\right)\approx 0$. In this case we obtain the following formula:

$$\frac{H_0-H_0^{Pl}}{H_0^{Pl}}\approx\frac{1}{2}\frac{k^\alpha k^\beta}{k^2}\left(\frac{h_{\alpha\beta}}{a^2}\right)_{z_g} \tag{6}$$

Let's consider the case when the galaxies belong to outskirts of the Local Supercluster. The average matter density contrast in superclusters is of order $\frac{\delta\varepsilon}{\varepsilon}\approx 0{,}1$ (see, for example, [11]). This density contrast corresponds to the metric perturbation $h_{\alpha\beta}$. In order of value $\frac{\delta\varepsilon}{\varepsilon}\approx\frac{k^\alpha k^\beta}{k^2}\left(\frac{h_{\alpha\beta}}{a^2}\right)_{z_g}$. Then the variation of the Hubble constant is $\frac{H_0-H_0^{Pl}}{H_0^{Pl}}\approx\frac{1}{2}\frac{\delta\varepsilon}{\varepsilon}\approx 0{,}05$. This value is comparable to the average variation of the Hubble constant in measurements [3].

The above example shows that variations of the Hubble constant can be used to study local gravitational perturbations.

In the fractal space-time is described the observer will find a variation of the Hubble constant like $\left(\frac{H_0-H_0^{Pl}}{H_0^{Pl}}\right)_m\approx\frac{q_m-1}{2}\cdot\frac{k^\alpha k^\beta}{k^2}\frac{\left(g_{\alpha\beta}\right)_m}{a^2}$. The values of the Hubble constant for a set of self-similar regions will form a fractal set $\left(H_0\right)_p\approx\left(1+\frac{q-1}{2}\frac{k^\alpha k^\beta}{k^2}\frac{\left(g_{\alpha\beta}\right)}{a^2}\right)^p\cdot H_0^{Pl}$

Conclusion

Here we consider the spatial variations of the Hubble constant according to Riess, et al. [3,4]. It is noted that the values of the Hubble constant form an almost fractal manifold. This fact suggests that the variations may be associated with local gravitational perturbations in the neighborhoods of galaxies, in which there are Cepheids and supernovae selected for measurement. It is shown that the spatial variations of the Hubble constant may be due to the fact that the galaxies belong to outskirts of the Local Supercluster.

The fractality of the Hubble constant spatial variations may be due to the fractal space-time. In this case sources are in regions of space-time that have self-similar geometric properties. The regions may be are the filaments of fractal cosmic web or self-similar regions with equals space-time intervals. The metric tensors of any two regions and their coordinates differ only by a constant coefficient.

Thus the variations of the Hubble constant can be used to study local gravitational perturbations and also fundamental properties of space-time.

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