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Short Communication

Sequential method conformal mappings

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Abstract

The well-known, very important Schwarz–Christoffel integral does not yet completely solve the problem of mapping a half-plane onto a predetermined polygon. This integral includes parameters (inverse images of the polygon), the relationship of which with the lengths of the edges of the polygon is not known in advance. The main difficulty in using the Schwarz–Christoffel formula lies in determining these parameters. If this difficulty can be overcome in some effective way, then the Schwarz–Christoffel formula expands the range of conformally mapped regions so much that it can be considered "universal", given that the curvilinear boundary of the mapped region can be approximated by a broken line. Thus, together with the Schwarz–Christoffel formula, a new direction in the theory of functions arises - numerical methods of conformal mapping. However, these approximate numerical methods were developed independently of the Schwarz–Christoffel formula.

Introduction

And yet, it has still not been possible to create a single simple and at the same time effective method for displaying any predetermined area.

Among the wide variety of proposed numerical conformal mapping methods, it is evident that they can be categorized into three groups: the boundary variation method, the trigonometric interpolation method, and the method by P.P. Kufarev and his developments [1-5].

Methods that involve varying the boundary of a region include the approach developed by P.F. Filchakov, which is based on a series of successive and appropriately selected elementary mappings. Therefore, it is natural to refer to this method as the method of successive conformal mappings or the exhaustion method.

Methodology

This method is detailed in [6-8], where applications related to the theory of filtration and groundwater flow beneath dams are also presented. Consider the following example, in which we need to map an infinite region (Z_0) onto the upper half-plane (W). This region (Z_0) also resides in the upper half-plane and is bounded below by a given contour. To achieve this, in the area where the contour of the region (Z_0) deviates the most from the real axis, we introduce an arc approximating an ellipse and associate a segment of the real axis with its boundary. Figure 1 illustrates the outcome obtained using the ellipse mapping formulas (1.1).

As illustrated in Figure 1, when an ellipse is mapped onto the upper half-plane, its boundary coincides with the real axis, and the contour of the region Z_0 approaches the real axis more closely. At this point, the contour resides within the Z_1 half-plane. By continuing the initial constructions for this region, we obtain a new approximation, and so forth. Implementing the ellipse mapping itself is most conveniently done in two successive stages, as per the following formulas:

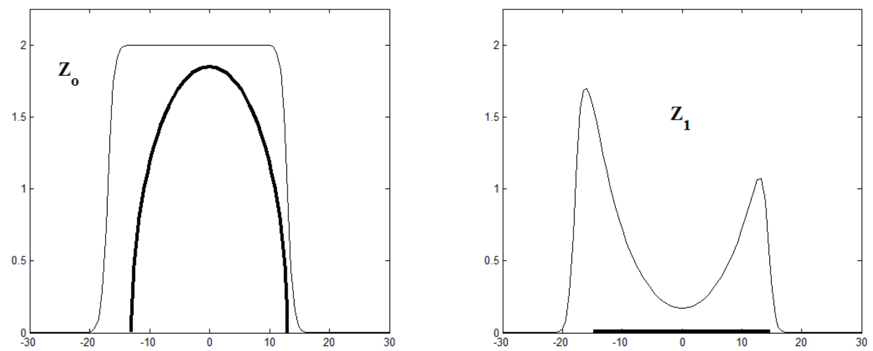

Figure 1: Mapping an ellipse onto the upper half-plane.

$$t = \frac{z_0 + \sqrt{z_0^2 - a^2 + b^2}}{a + b}, \qquad z_1 = \frac{a_1 + b_1}{2} \cdot t + \frac{a_1 + b_1}{2t} \tag{1.1}$$

The first equation in (1.1) defines the mapping from the exterior of an ellipse with given half-planes a and b to the exterior of a unit-radius circle. The second equation maps the exterior of a unit-radius circle to the exterior of another ellipse with half-planes a_1 and b_1. Additionally, the final ellipse can be degenerate, in which case one of the half-planes becomes zero, i.e., $b_{1} = 0$. In such instances, to maintain the normalization to infinity ($Z_0 \approx Z_1$), it is necessary to set $a_{1} = a + b$ or the value of the second half-axis.

Conclusion

Furthermore, it's worth noting the following properties of elliptic mappings:

Property 1: Points in the Z_n plane located on the half-ellipse and the real axis are mapped to points on the real axis in the Z_{n+1} plane.

Property 2: For all points in the half-planes Z_n and Z_{n+1}, the following inequalities hold: $| x_{n+1} + 1| > | x_n |$ and $| y_{n+1}| < | y_n |$.

By virtue of these observed properties, a sequence of elliptic mappings is capable of transferring any simply connected and univalent region in the Z plane to the upper half-plane W with any desired level of accuracy.

The development and modernization of conformal imaging are required for use in conformal mapping, electrostatics for calculating the distribution of electric fields, and continuum mechanics (hydraulic and aeromechanics, gas dynamics, theory of elasticity, theory of plasticity, etc.). Currently, the development of computational methods has led to the creation of compact models. matrix calculation technologies may be preferable for engineering use.

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