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Research Article

Mathematical modeling of velocity field induced by the vortex

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Abstract

In new technological applications, it is important to use vortex distributions in the area for obtaining large velocity fields. This paper, it was calculated the distribution of the velocity field and distribution of stream function for ideal incompressible fluid, induced by a different system of the finite number of vortex threads: 1) circular vortex lines in a finite cylinder, positioned on its inner, 2) spiral vortex threads, positioned on the inner surface of the finite cylinder or cone, and 3) linear vortex lines in the plane channel, positioned on its boundary.

An original method was used to calculate the components of the velocity vectors. Such kind of procedure allows calculating the velocity fields inside the domain depending on the arrangement, the intensity, and the radii of vortex lines. In this paper, we have developed a mathematical model for the process in the element of Hurricane Energy Transformer. This element is a central figure in the so-called RKA (ReaktionsKraftAnlage) used on the cars' roofs.

Introduction

The effective use of vortex energy in the production of strong velocity fields by the different devices is one of the modern areas of applications, developed during the last years, an example is the RKA (ReaktionsKraftAnlage) used on the cars' roofs for substations reducing the air's drag [1,2] (Figure 1), in the area for obtaining large velocity fields [3,4].

Figure 1: The cars' roofs.

In 2004 A.Bertasius, A.Buikis, and P.Verzbovicius formulated a patent [5] of apparatus and methods for heat generation. Later A.Buikis and H.Kalis have constructed a mathematical model of this heat generator [6-8].

In this model, the viscous electrically conducting incompressible liquid is located between two infinite coaxial cylinders (rings). The electromagnetic force drives magneto-hydrodynamic flow between the cylinders.

In 2009 designed a similar generator to [9] and created a mathematical model for the generator [10,11]. In the internal cylinder parallel to the axis are placed metal conductors-electrodes of the forms of bars. For those conductors, the alternating current is connected. The water is a weakly electrically conducting liquid (electrolyte). This is the mathematical model of one device for electrical energy produced by alternating current in the production of heat energy.

The distribution of electromagnetic fields, forces, 2D magnetohydrodynamic flow, and temperature induced by the

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system of the alternating electric current or external magnetic field in a conducting cylinder has been calculated using finite difference methods. An original method was used to calculate the mean values of electromagnetic forces.

The second interesting way in the vortexes exploitation in devices was collaboration with inventor J. Schatz in Germany [12,13]. In new technological applications, it is important to use vortex distributions for obtaining large values of velocity. The effective use of vortex energy in the production of strong velocity fields by different devices is one of the modern areas of applications, developed during the last decade. Such processes are ecologically clean; there is no environmental pollution. Although, on the other hand, the aspect of energy is very important: the transformation process should be organized in such a way that vortex energy is effectively transformed into heat or mechanical energy. In our previous papers [6,7,14,15] we have mathematically modeled the process how transforming the alternating electrical current into heat energy.

The practical aim of this investigation is to try to understand the process in the element of Hurricane Energy Transformer. This element is a central figure in the so-called RKA (German: ReaktionsKraft Anlage, English: Reaction Force Device) used on the cars' roof for substation reducing the airs' drag. This is all that's done at the practical level in mathematical modeling.

However, several practical and theoretical questions are left unanswered. Devices sometimes have worked with effectiveness higher than 100 Important is that in such a system there are strong vortices and electromagnetic fields or high velocities. For example, in [5,9] the alternating electro currency with voltage 380 V is about 1 ampere on 1 cm. Theoretically, the answer may be that we have a contradiction in the macro and micro processes in such devices [16].

Following Kim [17], we require a new paradigm beyond materialism including the information field on the theory of Physical vacuum. It is easy to call such science pseudoscience, but within its framework, it is possible to portray scalar waves [18,19]. In recent years, there have been several other new approaches: space-time as energy [19]. We should discuss these approaches with an open mind, without a simple rejection.

The goal of this paper is to develop mathematical models for new types of ecologically clean and energetically effective devices [12,20-23].

Such a type of device firstly was developed by I. Rechenberg [1]. Now the continuator of the work is one of the authors J. Schatz. The devices of such type can be considered as the energy source of the new generation. The practical aim of this investigation is to try to understand the process in the element of Hurricane Energy Transformer [12]. This element is the central figure in so so-called RKA (ReaktionsKraftAnlage) used on the cars' roof for substation reducing the air's drag.

This work presents three mathematical models of such devices. It is

1. a finite cylinder with a finite number of circular vortex lines positioned on its inner surface with a fixed distance between each other,
2. a finite cylinder or cone with a finite number of spiral vortex threads positioned on its inner surface,
3. a plane channel with a finite number of linear vortex lines positioned on its boundary.

It is well known that the vortex theory began from the Decart papers. First of all, it investigated the behavior of the discrete N linear vortex lines with an equal intensity Γ, which are in the vertices of the regular rectangle (authors are Helmholc, Kelvin, Kirhof, see [24-26]). The investigation of contemporary is written in the books [27,28]:

Completely are investigated linear vortex lines, vortex sheets, vortex wakes, vortexes of Karman, but difficulties cause the curves of vortex lines. In new technological applications, it is important to use vortex distributions for obtaining large values of velocity.

The mathematical model

Let the cylindrical domain (conus)

$$\Omega_{r,z}(\epsilon) = \{(r,z,\varphi): 0<r<a-\epsilon z, 0<z<Z, 0<\varphi<2\pi(M+1)\}\ (0 \le \epsilon Z < a)$$

contain ideal incompressible fluid,

where a, Z the maximal radius and length of the cylinder, M is the number of circulation periods.

If $\epsilon = 0$; then we have the circular cylinder with the radius a.

Consider the situation when the N discrete circular vortex lines

$$L_i = \{(r,z), r=a_i, z=z_i\}, 0<z_i<Z, 0<a_i<a,\}\ i=\overline{1,N},$$

with intensity $\Gamma_i(\frac{m^2}{s})$ and radii $a_i(m)$ are placed in the cylinder.

The vortex creates in the ideal compressible liquid the radial v_r and axial v_z components of the velocity field, which rises to the liquid motion.

Similar can be considered N discrete spiral vortex threads

$$S_i = \{(r,z,\varphi), r=a-\epsilon t, z=bt, \varphi=t+i\delta\}, i=\overline{1,N},$$

with parameters

$$\delta=\frac{2\pi}{N}, \tau=\frac{Z}{2\pi aM}, \frac{2\pi}{N} \le \varphi \le 2\pi(M+1), b=a\tau, t\in[0,2\pi M].$$

Here τ is the rise of the vortex threads, the spiral vortex with Z=2π, a=1, N=6, M=1, τ=1, ϵ=0;1.

In the Figure 2, we can see the circular vortex lines.

The spiral vortexes create in the ideal compressible liquid

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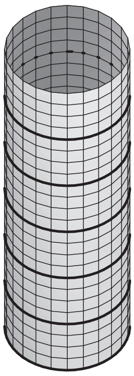

Figure 2: The surface of the cylinder with circular vortex lines.

the radial v_r, axial v_z, and azimuthal v_φ components of the velocity field.

The linear vortex lines create in the plane domain-channel

$$\Omega_{x,y} = \{(x,y): x \in [0,L], y \in [0,2], z \in (-\infty,\infty)\}$$

the v_x,v_y components of the velocity field.

The main aim of this work is to analyze the diversity of connection schemes of vortex curves that influence the maximal value of velocity.

Calculation of the velocity field for the spiral vortexes

The vector potential A is determined from the equations of vortex motion of ideal incompressible fluid [12,20,22,25,26]

$$div\, v = 0, rot\, v = \Omega,$$

in the following form:

$$\Delta A = -\Omega,$$

where v = $rotA$ and v,Ω the vectors of velocity and vortex fields are, Δ is the Laplace operator.

Applying the Biot-Savar law [25,26] we receive the following form of the vector potential created by the vortex thread W_i (W_i= S_i or W_i= L_i):

$$A(P)_i = \frac{\Gamma_i}{4\pi}\int_{W_i} \frac{dl}{R(QP)_i}$$

where dl is an element of the curves, P=$P(x,y,z)$ is the fixed point in the liquid, Q=$Q(\xi,\eta,\zeta)$ is the changeable point in the integral

$$R(QP)_i = \sqrt{((z-\zeta)^2 + (x-\xi_i)^2 + (y-\eta_i)^2)}.$$

From cylindrical coordinates x=$r\cos\varphi$, y=$r\sin\varphi$,

for the spiral vortexes S_i:

$$\xi_i = a_*(t)\cos(t+i\delta), \eta_i = a_*(t)\sin(t+i\delta), \zeta = bt, (b = a\tau),$$

$$t \in [0, 2\pi M]\ (a_*(t) = a - \epsilon t)$$

and we have the following components of the vector potential:

$$A_{x,i} = \frac{\Gamma_i}{4\pi}\int_{S_i}\frac{d\xi}{R_i}, A_{y,i} = \frac{\Gamma_i}{4\pi}\int_{S_i}\frac{d\eta}{R_i},$$

$$A_{z,i} = \frac{\Gamma_i}{4\pi}\int_{S_i}\frac{d\zeta}{R_i},$$

where $R_i = R(QP)_i$ (Figure 3).

Therefore

$$d\xi = (-a_*(t)\sin(t+i\delta) - \epsilon\cos(t+i\delta))dt,\ d\eta = (a_*(t)\cos(t+i\delta) - \epsilon\sin(t+i\delta))dt,\ d\zeta = bdt,$$

$$R_i = \sqrt{r^2 + a_*(t)^2 - 2a_*(t)r\cos(\varphi - t - i\delta) + (z-bt)^2}$$

and

$$A_{x,i} = -\frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{(a_*(t)\sin(t+i\delta) + \epsilon\cos(t+i\delta))dt}{R_i},$$

$$A_{y,i} = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{(a_*(t)\cos(t+i\delta) - \epsilon\sin(t+i\delta))dt}{R_i},$$

$$A_{z,i} = \frac{\Gamma_i b}{4\pi}\int_0^{2\pi M}\frac{dt}{R_i}.$$

The vector components of the velocity field (radial, axial, azimuthal) induced by the spiral vortex curves are in the form

$$\begin{cases} v_{r,i} = -\frac{\partial A_{\varphi,i}}{\partial z} + \frac{\partial A_{z,i}}{r\partial\varphi}, \\ v_{z,i} = \frac{1}{r}\frac{\partial}{\partial r}(rA_{\varphi,i}) - \frac{1}{r}\frac{\partial A_{r,i}}{\partial\varphi} \\ v_{\varphi,i} = \frac{\partial A_{r,i}}{\partial z} - \frac{\partial A_{z,i}}{\partial r}, \end{cases} \quad (1)$$

where

$$A_{r,i} = A_{x,i}\cos(\varphi) + A_{y,i}\sin(\varphi) = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{(a_*(t)\sin(\psi(t)) - \epsilon\cos(\psi(t)))dt}{R_i},$$

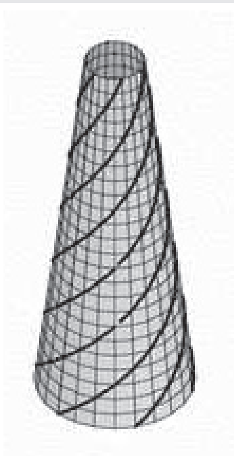

Figure 3: Spiral vortices on the cone with $\epsilon = -0.1$, $Z = 2\pi$.

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$$A_{\varphi,i} = -A_{x,i}\sin(\varphi) + A_{y,i}\cos(\varphi) = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{(a_*(t)\cos(\psi(t)) + \epsilon\sin(\psi(t)))dt}{R_i},$$

$$(\psi = \varphi - t - i\delta)$$

are the radial and azimuthal components of vector potentials.

Then from the partial derivatives

$$\frac{\partial R_i}{\partial r} = \frac{r - a_*(t)\cos(\psi(t))}{R_i}, \frac{\partial R_i}{\partial z} = \frac{z - bt}{R_i}, \frac{\partial R_i}{\partial \varphi} = \frac{a_*(t) r\sin(\psi(t))}{R_i},$$

follows

$$v_{r,i} = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{1}{R_i^3}[(z-bt)(a_*(t)\cos(\psi(t)) + \epsilon\sin(\psi(t))) - ba_*(t)\sin(\psi(t))]dt, \quad (2)$$

$$v_{z,i} = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{1}{R_i^3}[a_*(t)(a_*(t) - r\cos(\psi(t))) - \epsilon r\sin(\psi(t))]dt, \quad (3)$$

$$v_{\varphi,i} = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{1}{R_i^3}[b(r - a_*(t)\cos(\psi(t))) - (z-bt)(a_*(t)\sin(\psi(t)) + \epsilon\cos(\psi(t)))]dt.$$

For $\epsilon = 0$ and for the symmetrical properties respect to $z=z/2$ follows that for the all components of velocity $v_i(r, Z/2 - z, \varphi) = v_i(r, Z/2 + z, \varphi)$.

If r=0, then

$$v_{z,i}(0,z) = \frac{\Gamma_i}{4\pi}\int_0^{2\pi M}\frac{a_*(t)^2 dt}{(a_*(t)^2 + (z-bt)^2)^{1.5}} \quad (4)$$

or

$$v_{z,i}(0,z) = \frac{\Gamma_i \epsilon^2}{4\pi}\int_{a-2\pi M\epsilon}^{a}\frac{q^2 dq}{R(q)^3},$$

Where

$$R(q) = \sqrt{a_1 + b_1 q + c_1 q^2},\ a_1 = b^2 z_0^2, b_1 = -2b^2 z_0, c_1 = \epsilon^2 + b^2,\ z_0 = a - \frac{z\epsilon}{b}.$$

Therefore, from [24]:

$$\begin{cases} v_{z,i}(0,z) = \frac{\Gamma_i}{4c_1\pi}[\frac{d_2 a_2 - 2a_1 b_1}{d_1 R(a_2)} - \frac{d_2 a - 2a_1 b_1}{d_1 R(a)} - \\ \frac{\epsilon^2}{\sqrt{(c_1)}}\ln\frac{\sqrt{(c_1)}R(a_2) + c_1 a_2 + b_1/2}{\sqrt{(c_1)}R(a) + c_1 a + b_1/2}], \end{cases} \quad (5)$$

Where

$$a_2 = a - 2\pi\epsilon M,\ d_1 = 4b^2 z_0^2,\ d_2 = d_1(\epsilon^2 - b^2).$$

If ϵ=0, then

$$v_{z,i}(0,z) = \frac{\Gamma_i M}{2Z}[\frac{z}{\sqrt{a^2 + z^2}} + \frac{Z - z}{\sqrt{a^2 + (Z-z)^2}}], \quad (6)$$

and the maximal value of velocity is

$$v_{z,i}(0, Z/2) = \frac{\Gamma_i M}{2a\sqrt{1 + (Z/(2a))^2}} \quad (7)$$

by $z=Z/2$.

The minimal value we have in the form

$$v_{z,i}(0,0) = v_{z,i}(0,Z) = \frac{\Gamma_i M}{2a\sqrt{1 + (Z/a)^2}} \quad (8)$$

By z=0 and $z=Z$

The averaged value of the axial component of the velocity field in the axes of the cylinder (r=0) is

$$v_{av,i} = \frac{1}{Z}\int_0^Z v_{z,i}(0,z)dz. \quad (9)$$

The average value ϵ=0, r=0 is

$$v_{av,i} = \frac{\Gamma_i M}{2a}\frac{2}{1 + \sqrt{1 + (Z/a)^2}}. \quad (10)$$

From I. Rechenberg [1] (=0) in the middle point of finite vortex spool ($z=Z/2$) with the length Z the axial component of one vortex thread is

$$v_{max} = \frac{\Gamma_i}{\pi D}ctg(\beta)\sin(\arctan(\frac{Z}{D})) \quad (11)$$

where

β is the rise of vortex thread angles ($\beta=\arctan(\tau)$) and $D = 2a$ is the diameter of the vortex spool.

For the minimal value of velocity (in the points z=0 und $z=Z$) [1]:

$$v_{min} = \frac{\Gamma_i}{2\pi D}ctg(\beta)\sin(\arctan(\frac{Z}{a})). \quad (12)$$

We have equal values of v_{max} from (11) and from (7) using

$$\sin(\arctan(y)) = \frac{y}{\sqrt{1 + y^2}}, y = \frac{Z}{D},\ ctg(\beta) = \tau^{-1} = \frac{\pi DM}{Z}.$$

The average value (10) for ϵ=0 is in the following form

$$v_{av} = \frac{\Gamma_i}{\pi D}ctg(\beta)\frac{\alpha}{\alpha a/Z + 1}, \quad (13)$$

where $\alpha = \sin(\arctan(\frac{Z}{a}))$.

In the formulas parameters M and Z are depending:

$$M = \frac{Z}{\tau\pi D}, \tau = \tan(\beta).$$

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Therefore from (4) (13) for the velocity components (v_τ, v_z, v_φ) and the azimuthal component of the vector potential A_φ induced by N discrete vortex are

$$v_r = \sum_{i=1}^{N} v_{r,i}, v_z = \sum_{i=1}^{N} v_{z,i}, v_\varphi = \sum_{i=1}^{N} v_{\varphi,i}, A_\varphi = \sum_{i=1}^{N} A_{\varphi,i}. \quad (14)$$

Integrals are with the trapezoid formulas calculated.

If the intensity Γ_i of N – the spiral vortex S_i is equal Γ, then from (6) – (12) follows:

$$v_z(0, Z/2) = \frac{\Gamma NM}{D}\frac{1}{\sqrt{1+(Z/D)^2}}, \quad (15)$$

$$v_z(0,0) = v_z(0,Z) = \frac{\Gamma NM}{D}\frac{1}{\sqrt{1+(Z/a)^2}}, \quad (16)$$

$$v_{max} = \frac{\Gamma N}{\pi D} ctg(\beta)\sin(\arctan(\frac{H}{D})), \quad (17)$$

$$v_{min} = \frac{\Gamma N}{2\pi D} ctg(\beta)\sin(\arctan(\frac{H}{a})), \quad (18)$$

where N – the number of vortex threads, H=Z– and the height of the vortex spool (in building synonym of the length) are.

For the averaged value of velocity, (ϵ=0) we have the formula

$$v_{av} = \frac{\Gamma NM}{D}\frac{2}{1+\sqrt{1+(Z/a)^2}}, \quad (19)$$

or

$$v_{av} = \frac{\Gamma N}{\pi D}\frac{\alpha}{\alpha a/H+1} ctg(\beta), \quad (20)$$

where $\alpha = \sin(\arctan(\frac{H}{a}))$.

If the averaged value v_{av} is known, then it can be calculated from (19) also the dimensionless length $y = \frac{Z}{a}$ in the following form

$$y = \frac{2\delta}{\delta^2 - 1},$$

where

$$\delta = \Gamma N ctg(\beta)/(\pi D v_{av}).$$

An example, if $\Gamma = 6.0319(\frac{m^2}{s}), \beta = 10^0(C), D = 0.25(m),$

$N = 1, v_{av} = 30(\frac{m}{s}),$ then δ = 1.452 and y = 2.62, z=0.3275(m).

The corresponding formulas (15, 17); (16,18), and (19, 20) are identical, but from (15),(16), and (19) follows, that the velocity depending on the parameter $M*N$ is, where $M = \frac{H}{\tau\pi D}$.

From (15, 16) and (19) we can find the corresponding multiplicators by $\frac{\Gamma NM}{D}$ calculating (Table 1):

$$R_1 = \frac{1}{\sqrt{1+(Z/D)^2}}, R_2 = \frac{1}{\sqrt{1+(Z/a)^2}},$$

and

$$R_3 = \frac{2}{1+\sqrt{1+(Z/a)^2}}.$$

Calculation of the velocity field for the circular vortex lines

For the circular vortex lines:

$$\xi = a_i\cos\alpha, \eta = a_i\sin\alpha, \zeta = z_i, d\xi = -a_i\sin\alpha d\alpha,$$
$$d\eta = a_i\cos\alpha d\alpha, d\zeta = 0$$

and from axially-symmetric condition follows that by φ = 0 is $A_{x,i} = A_{z,i}$ = 0 and

$$A_{y,i} = A_{\varphi,i} = A_i(r,z) = \frac{\Gamma_i a_i}{4\pi} I_i,$$

where

$$I_i = \int_0^{2\pi}\frac{\cos\alpha d\alpha}{\sqrt{(z-z_i)^2 + a_i^2 + r^2 - 2a_i r\cos\alpha}}.$$

The integral ι_i is equal [25]

$$I_i = \int_0^{\pi/2}\frac{(1-2\sin^2 t)dt}{\sqrt{((z-z_i)^2+(r+a_i)^2)}\sqrt{1-k_i^2\sin^2 t}} = \frac{2}{\sqrt{ra_i}}[(\frac{2}{k_i}-k_i)K(k_i) - \frac{2}{k_i}E(k_i)],$$

where

$$t = (\alpha-\pi)/2, k_i = 2\sqrt{ar}/c_i, \quad c_i = \sqrt{(a_i+r)^2 + (z-z_i)^2},$$

$$K(k) = \int_0^{\pi/2}\frac{dt}{\sqrt{1-k^2\sin^2 t}}$$

is the total elliptical integral of the first kind,

$$E(k) = \int_0^{\pi/2}\sqrt{1-k^2\sin^2 t}dt$$

is the total elliptical integral of the second kind.

Therefore the azimuthal component of vector potential A_i induced by a circular vortex line L_i with intensity Γ, and radius a_i is

Table 1: Multiplicators of the velocity for vortexes by $\frac{Z}{a} = 1.4$.

N	$R_4(0)$	$R_4(Z/2)$	$R_4(Z)$	R_5	R_1	R_2	R_3
1	0.94	0.71	0.26	0.69	0.82	0.58	0.74
2	1.74	1.59	0.62	1.46	1.64	1.16	1.47
3	2.37	2.58	1.09	2.27	2.46	1.74	2.21
4	2.85	3.56	1.72	3.09	3.28	2.32	2.94
5	3.20	4.44	2.52	3.85	4.10	2.91	3.68
6	3.47	5.16	3.47	4.55	4.92	3.48	4.41

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$$A_i(r,z)=\frac{\Gamma_i}{2\pi}\sqrt{\frac{a_i}{r}}[(\frac{2}{k_i}-k_i)K(k_i)-\frac{2}{k_i}E(k_i)].$$

The vectorial components of the velocity field (the radial and axial components) induced by the vortex line L_i are

$$v_{r,i}=-\frac{\partial A_i}{\partial z}, v_{z,i}=\frac{1}{r}\frac{\partial}{\partial r}(rA_i). \quad (21)$$

or

$$v_{r,i}(r,z)=\frac{\Gamma_i}{2\pi r}\frac{z-z_i}{c_i}[E(k_i)\frac{a_i^2+r^2+(z-z_i)^2}{(a_i-r)^2+(z-z_i)^2}-K(k_i)], \quad (22)$$

$$v_{z,i}(r,z)=\frac{\Gamma_i}{2\pi c_i}[K(k_i)+\frac{a_i^2-r^2-(z-z_i)^2}{(a_i-r)^2+(z-z_i)^2}E(k_i)]. \quad (23)$$

If $r = 0$ then

$$v_{z,i}(0,z)=\frac{\Gamma_i}{2}\frac{a_i^2}{(a_i^2+(z-z_i)^2)^{1.5}}. \quad (24)$$

This component of vectors has the maximal value $v_{z,i}=\frac{\Gamma_i}{2a}$ by $z=z_i$, $a_i=a$.

By $z=z_i+Z/2$ we have

$$v_{z,i}=\frac{\Gamma_i}{2\sqrt{a^2+Z^2/4}}\frac{a^2}{a^2+Z^2/4}<\frac{\Gamma_i}{2\sqrt{a^2+Z^2/4}}-$$

this is the value of the component of velocity induced by a spiral vortex (ϵ=0).

If $z=Z/2$, $a_i=a$ then from (24) follows

$$v_{z,i}(0,Z/2)=\frac{\Gamma_i}{D}\frac{1}{(1+((Z/2-z_i)/a)^2)^{1.5}}. \quad (25)$$

For the averaged value of the velocity we have

$$v_{av,i}=\frac{\Gamma_i}{D}\frac{a}{Z}(\frac{(Z-z_i)/a}{\sqrt{1+((Z-z_i)/a)^2}}+\frac{z_i/a}{\sqrt{1+(z_i/a)^2}}). \quad (26)$$

If $z_i = Z/2$, then

$$v_{av,i}=\frac{\Gamma_i}{D}\frac{1}{\sqrt{1+(Z/D)^2}}.$$

The summary velocity field (v_r,v_z) and the vector potential $A\varphi$ induced by N discrete vortex lines we obtained in the form (14). The hydrodynamic stream function $\psi = \psi(r,z)$ for velocity components

$v_r=-\frac{1}{r}\frac{\partial\psi}{\partial z}$, $v_r=\frac{1}{r}\frac{\partial\psi}{\partial r}$, from (21) is $\psi(r,z)=rA_\varphi(r,z)$.

The amount of flow through the cross-section $[z=z_0,0<r<a_0]$ is

$$Q(a_0,z_0)=\int_0^{a_0}\int_0^{2\pi}v_z(r,z_0)rdr\,d\varphi=2\pi a_0A_\varphi(a_0,z_0)=2\pi\psi(a_0,z_0).$$

The total amount of flow through cross cylindrical domain $[0<z<Z,0<r<a_0]$is

$$Q_t(a_0)=\int_0^Z Q(a_0,z)dz=2\pi\int_0^Z\psi(a_0,z)dz.$$

For the circular vortex line, if $z_i/a=0.2i, i=\overline{1,N}, N\le 6,$ we can calculate the following multiplicators by the factor $\frac{\Gamma}{D}$:

$$R_4(Z)=\sum_{i=1}^{N}(1+((Z/2-z_i)/a)^2)^{-1.5}$$

for (25),

$$R_5=\frac{a}{Z}\sum_{i=1}^{N}(\frac{(Z-z_i)/a}{\sqrt{1+((Z-z_i)/a)^2}}+\frac{z_i/a}{\sqrt{1+(z_i/a)^2}})$$

for (26).

An example, if Z/a = 1.4 then we can the multiplicators

$R_4((0)$, $R_4(Z/2)$,$R_4(Z)$,R_5 for the circular vortex lines and R_1,R_2,R_3 for the spiral vortexes, by the factor $\frac{\Gamma M}{D}$ in the form R_1*N R_2*N R_3*N calculated (Table 1).

In the following calculations we use the dimensionless form scaling all the lengths to r_o = a (the inlet radius of the tube), the axial v_z and radial v_r velocity to $v_0=\frac{\Gamma_0}{2\pi r_0}$, the azimuthal components of vector potential $A\varphi$ to $A_0=\frac{\Gamma_0}{2\pi}$, the stream function ψ to $\psi_o=A_or_o$ and the total amount of flow Q to $Q_o={}_or_o$. Here Γ_o is dimensional scaling of vortex intensity $\Gamma_i, i=\overline{1,N}$.

The flow field induced by linear vortex lines in a channel

Unlike our previous papers [20,21] here we additionally consider the chain of linear vortex lines in the plane channel. For symmetry-conditions, $\frac{\partial v_x}{\partial y)}|_{y=1}$ we consider half the plane channel $y\in[0,1]$.

In the plane, y=0 we have the slip conditions v_x=v_y=0 for the velocity vectors of viscous incompressible liquid.

The flow in the channel is given by a fixed amount of flow through a cross-section of the half-channel $Q=\int_0^1 v_x|_{x=0}\,dy$.

If $L=\infty$, then $v_x=u(y)$,v_y = 0 we have the Puaseil flow $u = Q(3y-1.5y^2)$ – the solution of Navier-Stokes equation in the channel $\Omega_{x,y}$.

The wall y=0 of the channel is placed in a linear chain of vortexes with the axis transfer of the (x,y) plane. The one linear vortex line in the point (x_k,y_k)) creates the following components of velocity:

$$v_x=-\frac{\Gamma_k}{2\pi}\frac{y-y_k}{R^2}, v_y=\frac{\Gamma_k}{2\pi}\frac{x-x_k}{R^2}, \quad (27)$$

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where $R^2 = (x - x_k)^2 + (y - y_k)^2$.

In the center of this point-wise vortex, the velocity field is infinite therefore we consider the vortex line with the finite cross-section

the circle with radius a. In this case the expressions (27) are valid when $R \geq a$. but for $R<a$ we have

$$v_x = -\frac{\Gamma_k}{2\pi a^2}(y - y_k), v_y = \frac{\Gamma_k}{2\pi a^2}(x - x_k). \quad (28)$$

Some numerical results and discussion

The flow in the channel

We consider the channel with finite length L = 2.5, Puaseil flow with Q=3 and three wise of the chain of vortexes:

1) the main chain with coordinates and radius of the linear vortex

$$x_k = 0.2 + (k-1)0.4, y_k = 2a, k = 1,2,3,4,5,6, a = 0.05, \quad (29)$$

rotate clockwise with the intensity Γ_1,

2) the second chain with coordinates and radius of the linear vortex

$$x_k = 0.4 + (k-1)0.4, y_k = 2a_1, k = 1,2,3,4,5, a_1 = 0.025, \quad (30)$$

rotate opposite clockwise with the intensity Γ_2,

3) the thread chain with coordinates and radius of the linear vortex

$$x_k = 0.3 + (k-1)0.4, y_k = 2a + a_1, k = 1,2,3,4,5, a = 0.05, a_1 = 0.025, \quad (31)$$

rotate opposite clockwise with the intensity Γ_3.

For the pointwise vortexes line (29) outside the channel (y_k = -0.025) Γ_1 = -6 we have the following results: mV = 5.9895, mX = 1.00, mY = 0.

For the Karman chain [25] of vortexes (preliminary vortexes line and (30) $(y_k = -0.05, \Gamma_2 = 6)$ we have mV = 3.9790, mX = 0.20, mY = 0.

In the following Table 2 can see the amount (Q), maximal value of velocity u,(mv) with the coordinates (mX,mY) depending on the vortex intensity Γ_1, Γ_2, Γ_3

The circular vortexes lines

The basis for the calculations of N circular vortex lines $L_i, i = \overline{1,N}$ are $N \leq$ 6 chosen, which are arranged in the axial direction at the points with the following dimensionless coordinates $(z_i = 0.2i, r_i = a_i), i = \overline{1,N}$.

The dimensionless radius of the circular vortex lines a_i is considered in three forms (the sequence $a = [a_1, a_2, a_3, a_4, a_5, a_6,]$):

1. the constant sequence(radius of the cylinder) a_c = [1,1,1,1,1,1],
2. the monotonous increasing sequence a_{in} = [.75,.80,.85,.90,.95,1.0],
3. the monotonous decreasing sequence a_d = [1.0,.95,.90,.85,.75],

The results of numerical experiments for dimensionless values v_r, v_z, ψ, Q_t was obtained of different dimensionless intensity of vortex lines

$$\tilde{\Gamma}_i = \frac{\Gamma_i}{2\pi\Gamma_0} = \pm 6; \pm 3; \pm 2; 1; 0.5, \text{ and } l = Z / r_0 = 2, a_0 = 0.7.$$

The summary intensity of absolute values is equal to 6.

The velocity field is calculated on the uniform grid ($n_r \times n_z$) by the steps h_1=h_2=0.1 in the r,z directions.

The numerical results show that the velocity field induced by circular vortex lines is concentrated inside the cylinder. The results depend on the arrangement and the radius of vortex lines a_i.

Typical results of calculations are the dimensionless velocity field and the distribution of stream function in the cylinder. We can see the velocity formation depends on the arrangement of vortice lines with coordinates zj=[z_1,z_2,z_3,z_4,z_5,z_6], and of the radii a_i.

If $\tilde{\Gamma}_i > 0$ then all vortices move in the positive direction of Oz axis (v_z>0), but the radii of vortex lines to stay a different way (for v_r<0 the radius is decreasing and for v_r>0 the radius is increasing).

We obtain the dimensionless values of

$$v_r \in [v_{r.min}, v_{r.max}], \ vz_{max}, \psi_{max}, Q_t$$

for zj = [0.2,0.4,0.6,0.8,1.0,1.2] and for different radius of vortex lines a_i and sequence of intensity gj=[g_1,g_2,g_3,g_4,g_5,g_6] the following results:

Table 2: The dependence of flow velocity on the intensity of the vortexes.

Γ_1	Γ_2	Γ_3	Q	mV	mX	mY
0	0	0	3.00	4.500	0.00	1.00
-6	3	3	3.97	18.19	2.20	0.15
-6	4	4	3.46	22.90	0.30	0.10
-6	3	0	4.62	18.36	0.20	0.15
-6	2	2	4.49	18.63	2.20	0.15
-6	1	1	5.00	19.08	2.20	o.15
-6	1	0	5.22	19.14	0.20	0.15
-6	0	1	5.30	19.47	2.20	0.15
-6	0	0	5.52	19.86	1.00	0.15

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1. The radii are constant a_c = [1,1,1,1,1,1]

1.1 The intensity of the one vortex line L_3 is $\tilde{\Gamma}_3 = 6, N = 1$:

$v_r \in (-5.9, 5.9)$, vz_{max} = 18.85, ψ_{max} = 3.25,

v_r = 0 if $z=z_3$ = 0.6 and $v_r > o$ if $z > z_3$, therefore the radius of the vortex increased [26];

1.2 The intensity of the one vortex line L_3 is $\tilde{\Gamma}_3 = -6, N = 1$ (in the opposite direction):

$v_r \in (-5.9, 5.9)$, vz_{max} = -18.85, ψ_{max} = - 3.25,

the vortex moves in the negative direction of the Oz axes ($v_z<0$), v_r = 0 if $z=z_3$ = 0.6 and v_r > 0 if $z<z_3$, therefore the radius of the vortex also increases [26];

1.3 The intensity of the two vortex lines L_3, L_4 are $\tilde{\Gamma}_3 = 3, \tilde{\Gamma}_4 = 3, N = 2$:

$v_r \in (-5.7, 5.7)$, vz_{max} = 18.57, ψ_{max} = 3.17,

the vortexes move in the positive direction of Oz axes ($v_z>0$), v_r = 0 if $z=(z_3+z_4)/2=0.7$ and $v_r(a_o,z_3)$ = -2.46, $v_r(a_o,z_4)$ = 4.37, therefore the radius of the first vortex lines L_3 decreased, but for the second vortex lines L_4 increased and the first vortex can be move through the second vortex [26];

1.4 The intensity of the two vortex lines L_3, L_4 are $\tilde{\Gamma}_3 = -3, \tilde{\Gamma}_4 = 3, N = 2$:

$v_r \in (-2.9, 0.64)$, $v_z \in (-3.0, 3.0)$, $\psi \in (-0.32, 0.32)$,

v_z = 0 if z = 0.7 and $v_z(a_o,z_3)$=-1.72, $v_z(a_o,z_4)$ = 2.76, therefore the first vortex moves to the negative direction, but the second to the positive direction of Oz axes and the radii of the vortexes decreased (this case is in [26] considered);

1.5 The intensity of the two vortex lines L_3, L_4 are $\tilde{\Gamma}_3 = 3, \tilde{\Gamma}_4 = -3, N = 2$:

$v_r \in (-0.64, 2.9)$, $v_z \in (-3.0, 3.0)$, $\psi \in$ (-0.32, 0.32),

v_z = 0 if z = 0.7 and $v_z(a_o,z_3)$=1.72, $v_z(a_o,z_4)$ = -2.76, the first vortex moves to the positive direction, but the second to the negative direction of Oz axes and the radius of the vortexes increases [26];

1.6 The intensity of the three vortex lines L_1, L_3, L_5 are $\tilde{\Gamma}_1 = 2, \tilde{\Gamma}_3 = 2, \tilde{\Gamma}_5 = 2, N = 3$:

$v_r \in (-4.1, 4.1)$, vz_{max} = 16.34, ψ_{max} = 2.63,

v_r = 0 if $z=z_3$ = 0.6 and $v_z(a_0,z_1)$=15.92, $v_z(a_0,z_3)$=16.16, $v_z(a_0,z_5)$=15.92, $v_r(a_0,z_1)$=-3.8, $v_r(a_0,z_5)$= 1.6, the vortexes move in the positive direction of Oz axis and the radius of the first vortex decreased, but of the third vortex increased;

1.7 The intensity of the three vortex lines L_1, L_3, L_5 are $\tilde{\Gamma}_1 = -2, \tilde{\Gamma}_3 = 2, \tilde{\Gamma}_5 = -2, N = 3$:

$v_r \in (-1.6, 1.6)$, vz_{min} = -5.83, ψ_{min} = -0.74,

v_r = 0 if $z=z_3$=0.6, z=0.1, z=1.1 and $v_z(a_0,z_1)$=-5.67, $v_z(a_0,z_3)$=-2.42, $v_z(a_0,z_5)$=-3.56, $v_r(a_0,z_1)$=-0.77, $v_r(a_0,z_5)$=0.77, the vortexes move in the negative direction of Oz axis and the radius of the first vortex decreased, but of the third vortex increased;

1.8 The intensity of the three vortex lines L_1, L_3, L_5 are $\tilde{\Gamma}_1 = 2, \tilde{\Gamma}_3 = -2, \tilde{\Gamma}_5 = 2, N = 3$:

$v_r \in (-1.6, 1.6)$, $v_{z.max}$ = -5.83, ψ_{max} = 0.74,

v_r = 0 if $z=z_3$=0.6, and $v_z(a_0,z_1)$=5.67, $v_z(a_0,z_3)$=1.97, $v_z(a_0,z_5)$=5.67, $v_r(a_0,z_1)$=0.77, $v_r(a_0,z_5)$=-0.77, the vortexes move in the positive direction of Oz axis and the radius of the first vortex increased, but of the third vortex decreased;

1.9 The intensity of the three vortex lines L_1, L_3, L_5 are $\tilde{\Gamma}_1 = -2, \tilde{\Gamma}_3 = 2, \tilde{\Gamma}_5 = 2, N = 3$:

$v_r \in (-4.9, 2.6)$, $v_z \in (-1.75, 11.1)$, $\psi \in (-0.10, 1.45)$,

v_r = 0 if z= 0.9 and $v_z(a_0,z_1)$=-0.64, $v_z(a_0,z_3)$=8.28, $v_z(a_0,z_5)$=10.89, $v_r(a_0,z_1)$=-3.17, $v_r(a_0,z_3)$=-3.95, $v_r(a_0,z_5)$= 0.77, the two vortexes L_3, L_5 move in the positive direction, but the first in the negative direction of Oz axis and the radii of the two vortexes L_1, L_3 are decreased, but of the third vortex increased;

1.10 The intensity of the three vortex lines L_1, L_3, L_5 are $\tilde{\Gamma}_1 = 2, \tilde{\Gamma}_3 = 2, \tilde{\Gamma}_5 = -2, N = 3$:

$v_r \in (-2.6, 4.9)$, $v_z \in (-1.75, 11.1)$, $\psi \in (-0.10, 1.45)$,

v_r = 0 if z = 0.3 and $v_z(a_0,z_1)$= 10.89, $v_z(a_0,z_3)$=8.28, $v_z(a_0,z_5)$=-0.64, $v_r(a_0,z_1)$=-0.77, $v_r(a_0,z_3)$= 3.95, $v_r(a_0,z_5)$= 3.17, the two vortexes L_1, L_3 move in the positive direction, but the vortex L_5 in the negative direction of Oz axis and the radii of the two vortexes L_3, L_5 are increased, but of the third vortex L_1 decreased;

1.11 The intensity of the three vortex lines L_1, L_3, L_5 are $\tilde{\Gamma}_1 = -2, \tilde{\Gamma}_3 = -2, \tilde{\Gamma}_5 = 2, N = 3$:

$v_r \in (-4.9, 2.6)$, $v_z \in (-11.1, 1.75)$, $\psi \in (-1.45, 0.10)$,

v_r = 0 if z = 0.3 and $v_z(a_0,z_1)$= -10.89, $v_z(a_0,z_3)$=-8.28, $v_z(a_0,z_5)$=0.64, $v_r(a_0,z_1)$=0.77, $v_r(a_0,z_3)$=-3.95, $v_r(a_0,z_5)$= -3.17, the two vortexes L_1, L_3, move in the negative direction, but the third in the positive direction of Oz axis and the radii of the two vortexes L_3, L_5 are decreased, but of the first vortex increased.

2. The radii are increasing a_{in},

2.1 The non-uniform distribution of intensity gj =

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[2,2,1,.5,.5,0], N=5:

$v_r \in (-12.7, 7.4)$, $v_{z.max}$ = 21.15, ψ_{max} = 4.6, Q_t = 28.34,

v_r = 0 if z = 0.3, the radius of the first vortex decreased, but increased the radii of the last four vortexes ;

2.2 The distribution of intensity gj = [2,2,2,0,0,0], N=3:

$v_r \in (-13.3, 10.02)$, $v_{z.max}$ = 22.23, ψ_{max} = 4.8, Q_t = 28.69,

v_r = 0 if z = 0.3, the radius of the first vortex decreased, but increased the radii of the last vortex ;

2.3 The distribution of intensity gj = [0,0,3,3,0,0], N=2:

$v_r \in (-10.2, 9.4)$, $v_{z.max}$ = 21.14, ψ_{max} = 4.64, Q_t = 29.20,

v_r = 0 if z = 0.7, the radius of the first vortex decreased, but increased the radii of the last vortex ;

2.4 The intensity of the first vortex lines gj = [6,0,0,0,0,0], N=1:

$v_r \in (-19.2, 19.2)$, $v_{z.max}$ = 25.13, ψ_{max} = 6.47, Q_t = 27.10,

v_r = 0 if z = 0.2, the radius of the vortex increased;

2.5 The intensity of second vortex lines gj = [0,6,0,0,0,0], N=1:

$v_r \in (-15.0, 15.0)$, $v_{z.max}$ = 23.56, ψ_{max} = 5.69, Q_t = 29.28,

v_r = 0 if z = 0.4, the radius of the vortex increased;

2.6 The intensity of third vortex lines gj = [0,0,6,0,0,0], N=1:

$v_r \in (-11.8, 11.8)$, $v_{z.max}$ = 22.18, ψ_{max} = 5.11, Q_t = 29.69,

v_r = 0 if z = 0.3, the radius of the vortex increased;

2.7 The intensity of fourth vortex lines gj = [0,0,0,6,0,0], N=1:

$v_r \in (-9.6, 9.6)$, $v_{z.max}$ = 20.94, ψ_{max} = 4.66, Q_t = 28.72,

v_r = 0 if z = 0.3, Hence, the radius of the vortex increased.

3. The uniform distribution of intensity gj = [1,1,1,1,1,1]

3.1 Radii of vortex lines are constant (the sequence a_c:

$v_r \in (-4.5, 4.5)$, $v_{z.max}$ = 16.21, ψ_{max} = 3.14, Q_t = 25.12,

v_r = 0 if z = 0.7, the radii of the first three vortexes decreased, but of the last three vortexes increased ;

3.2 Radii of vortex lines are a_{in}:

$v_r \in (-8.4, 4.9)$, $v_{z.max}$ = 17.98, ψ_{max} = 3.52, Q_t = 27.36,

v_r = 0 if z = 0.8 the radii of the first three vortexes decreased but of the last two vortexes increased;

3.3 Radii of vortex lines are a_d:

$v_r \in (-4.9, 8.4)$, $v_{z.max}$ = 17.98, ψ_{max} = 3.52, Q_t = 27.36,

v_r = 0 if z = 0.5 the radii of the first two vortexes decreased but increased the radii of the last four vortexes.

4. The distribution of intensity gj = [2,2,.5,.5,.5,.5]

4.1 Radii of vortex lines are a_{in}:

$v_r \in (-12.3, 6.9)$, $v_{z.max}$ = 20.19, ψ_{max} = 4.4, Q_t = 27.77,

v_r = 0 if z = 0.3, the radius of the first vortex decreased but increased the radii of the last five vortexes;

4.2 Radii of vortex lines are a_d:

$v_r \in (-5.7, 5.6)$, $v_{z.max}$ = 17.30, ψ_{max} = 3.4, Q_t = 26.0.1,

v_r = 0 if z = 0.4, the radius of the first vortex decreased but increased the radii of the last four vortexes.

5. The distribution of intensity gj = [.5,.5,.5,.5,.5,2,2]

5.1 Radii of vortex lines are a_{in}:

$v_r \in (-5.6, 5.8)$, $v_{z.max}$ = 17.30, ψ_{max} = 3.4, Q_t = 26.0.1,

v_r = 0 if z = 1.0, the radii of the first four vortexes decreased but increased the radius of the last vortex;

5.2 Radii of vortex lines are a_d:

$v_r \in (-6.8, 12.3)$, $v_{z.max}$ = 20.19, ψ_{max} = 4.4, Q_t = 27.7.7,

v_r = 0 if z = 1.1, the radii of the first five vortexes decreased, but those of the last vortex increased.

6. The distribution of intensity gj = [.5,.5,2,2,.5,.5]

6.1 Radii vortex lines are a_{in}:

$v_r \in (-7.4, 6.6)$, $v_{z.max}$ = 19.47, ψ_{max} = 4.0, Q_t = 28.28,

v_r = 0 if z = 0.7 the radii of the first two vortexes decreased but increased the radii of the last four vortexes;

6.2 Radii of vortex lines are a_d:

$v_r \in (-6.6, 7.4)$, $v_{z.max}$ = 19.47, ψ_{max} = 4.0, Q_t = 28.28,

v_r = 0 if z = 0.7 the radii of the first two vortexes decreased but increased the radii of the last four vortexes.

6.3 The spiral vortexes in the cylinder (ϵ=0)

We consider $N \le 6$ spiral vortexes $S_i, i=\overline{1,N}$, which started from the points (a,0,$i2\pi/N$) at the cylinder.

The dimensionless radius of the cylinder a is equal to 1.

All results of the numerical experiments are for the dimensionless values $A_\varphi(a_o,z,\varphi), v_z(0,z), Q(z), Q_t$ and parametern $l=Z/a$ =0.5;1;1.5;2;3, a_o = 0.7 obtained.

The summary intensity of absolute values is equal to 6.

The azimuthal components of the vector potential are in the uniform grid ($N_z \times N_\varphi$) by the steps $h_z = l/N_z$, $h_\varphi = 2\pi/N_\varphi$,($N_z$=

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$N_\varphi = 30$) in the r,φ direction calculed.

The component $A_\varphi(z,\varphi)=(r=a_0)$ using the trapezoid formula is calculated. Figures show typical results of calculations: the dimensionless velocity field and the distribution of the azimuthal component of the velocity ($r=a_0$) in the cylinder.

The velocity formation depends on the length / of the cylinder.

The maximum of the azimuthal components of vector potentials A_{max} is depending of the intensity parameter $g_i = \tilde{\Gamma}_i$.

We obtain the dimensionless values of $v_{z.max}, Q_{max}, A_{max}, Q_t$, and for different sequence of intensity $gj=[g_1,g_2,g_3,g_4,g_5,g_6]$ the following results:

1. The length is ι =1.5,

($v_{z.max}$ = 15.08, Q_{max} = 24.98, Q_t = 33.20)

1. The uniform distribution of the intensity

gj = [1,1,1,1,1,1], N=6: A_{max} = 5.68,

the distribution A_φ is uniform in the φ direction;

2. The distribution of the intensity is gj = [2,2,1,.5,.5,0], N = 5:

A_{max} = 5.68, the distribution of A_φ is nonuniform in the φ direction;

3. The distribution of the intensity is gj = [2,2,2,.0,0,0], N = 3:

A_{max} = 5.75, the values of A_φ oscillate in the φ direction;

4. The distribution of the intensity is gj = [2,1,1,1,1,0], N = 5:

A_{max} = 6.14, the distribution of A_φ is nonuniform in the φ direction; (the maximal value 6.14 is in the point (0.75,4.2);

5. The distribution of the intensity is gj = [1.5,1.5,1.5,1.5,0,0], N = 4:

A_{max} = 5.68, the values of A_φ weakly oscillate in the φ direction;

6. The distribution of the intensity is gj = [3,3,0,0,0,0], N = 2:

A_{max} = 5.83, the distribution of A_φ is nonuniform in the φ direction with 3 maximums;

7. The distribution of the intensity is gj = [6,0,0,0,0,0], N = 1:

A_{max} = 8.54, the distribution of A_φ is nonuniform in the φ direction with one maximum;

8. The distribution of the intensity is gj = [6,0,0,0,0,0], N = 1, b=0;

A_{max} = 8.40, $v_{z.max}$ = 18.85, Q_{max} = 36.95, Q_t = 24.54 the distribution A_φ is uniform in the φ direction (this is the velocity field induced by the circular vortex line (z_i = 0)).

2. Different lengths ι

1. The distribution of the intensity is gj = [6,0,0,0,0,0], N = 1:, l=1.

A_{max} = 8.42, $v_{z.max}$ = 18.29, Q_{max} = 36.25, Q_t = 16.28;

2. The distribution of the intensity is gj = [2,2,1,.5,.5,0], N = 5, ι =3:

A_{max} = 5.11, $v_{z.max}$ = 10.46, Q_{max} = 16.36, Q_t = 43.03;

3. The distribution of the intensity is gj = [2,2,1,.5,.5,0], N = 5, ι =2:

A_{max} = 6.0386, $v_{z.max}$ = 13.3285, Q_{max} = 21.4252, Q_t = 37.6009,

If $N_{\varphi} = N_z = M$ = 50, then A_{max} = 6.0386, $v_{z.max}$ = 13.3286, Q_{max} = 21.4252, Q_t = 37.6017;

1. The distribution of the intensity is gj = [2,2,1,.5,.5,0], N = 5, ι =1:

A_{max} = 7.35, $v_{z.max}$ = 16.86, Q_{max} = 29.39, Q_t = 26.65,

5. The distribution of the intensity is gj = [2,2,1,.5,.5,0], N = 5, ι = 5:

A_{max} = 8.11, $v_{z.max}$ = 18.29, Q_{max} = 34.25, Q_t = 16.28,

6.4 The spiral vortexes in the cones ($\epsilon \neq 0$)

In this case, we have some results for the behavior of spiral vortexes.

1. If

$\Gamma = 6.0319(m^2/s), N=1, \beta = 10^0(C)(\tau = tg(\beta) = 0.1763),\ a = 0.125(m), Z \in [0.1,1.0](m),$

then from the formulas (14, 18) can be the values $M; V_1(\epsilon=0);\ V_2(\epsilon=0.001);\ V_3(\epsilon=0.002);\ V_4(\epsilon=-0.002)(m/s)$ calculated (Table 3).

For V_2 and V_3 the radii by $Z=1$ decreased from $a=0.125(m)$ with $0.080(m)$ and $0.034(m)$, but for V_4 the radius increased with $0.216(m)$.

2. If $a=0.25(m)$, then similar to the formulas (9, 20) can be the values $M; V_1(\epsilon=0);\ V_2(\epsilon=0.004);\ V_3(\epsilon=0.008);$ $V_4(\epsilon=-0.008)(m/s)$ calculated (Table 4).

For v_2 and v_3 the radii by z=1 decreased from a=0.25(m) with 0.16(m) and 0.07(m), but for v_4 the radius increased with 0.43(m).

Conclusion

1. Velocity fields of ideal compressible fluid influenced by a curved vortex field in a finite cylinder, finite cone, and channel are investigated.

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Table 3: The velocity V_{av} by a = 0.125.

Z	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
M	0.72	1.44	2.17	2.89	3.61	4.33	5.06	5.78	6.50	7.22
V_1	15.3	24.1	29.0	32.0	34.0	35.4	36.5	37.3	37.9	38.5
V_2	15.5	24.6	29.7	32.7	34.8	36.2	37.3	38.2	38.8	39.4
V_3	15.7	25.1	30.3	33.5	35.6	37.1	38.2	39.1	39.8	40.4
V_4	14.9	23.3	27.9	30.7	32.6	33.9	34.9	35.7	36.3	36.8

Table 4: The velocity V_{av} by a = 0.25.

Z	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
M	0.36	0.72	1.08	1.44	1.80	2.17	2.53	2.89	3.25	3.61
V_1	4.19	7.64	10.2	12.1	13.5	14.5	15.4	16.0	16.6	17.0
V_2	4.27	7.86	10.6	12.6	14.0	15.2	16.0	16.7	17.3	17.8
V_3	4.34	8.10	11.0	13.1	14.6	15.9	16.8	17.6	18.2	18.7
V_4	4.06	7.23	9.54	11.2	12.4	13.4	14.1	14.7	15.2	15.6

2. Numerical results show that the maximum axial velocity and the total amount of flow depend on the connection method of producers of vortex energy.

3. The maximal velocity is developed in the case of non-uniform distribution of vortex intensity and smaller radius of vortex lines.

4. The maximal value of the velocity induced by the spiral vortexes is in the middle of the cylinder.

5. The behavior of vortex lines in the ideal incompressible flow depends on the number and the orientation of the vortex.

6. The realization of circular vortices inside the pipe at the surface accelerates the flow speed inside the pipe if they rotate clockwise the flow depends on the values of parameters Re, Γ, A and the inflow mode in the pipe.

7. The calculations are related to specific applications of vortices in energy [21,23,26,25.

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